3-Dimensional VLSI Routing

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Contents

1 Introduction

This paper studies the design of very large scale integrated circuits. While many problems in this field are NP-complete, some of them can be handled by combinatorial algorithms. One of the main areas of very large scale integrated routing is called detailed routing. As for this type of routing, the devices of an electric equipment are placed on the four boundaries of a rectangular board. The aim is to connect certain given subsets of devices by wires. To do this we are allowed to use a rectangular grid that contains layers parallel with the rectangular board which contains the terminals on its boundary. In former times technology allowed only a very limited number of layers to use, that is why detailed routing was considered a planar, 2-dimensional problem.

Recently, thanks to the technological developments we are permitted to use more and more layers. This paper studies the single active layer routing problem. The problem itself is 3-dimensional, the devices of an electric equipment are placed on a rectangular circuit board. Again, we are supposed to connect certain given subsets of the devices by wires and this routing is to be realized in a 3-dimensional cubic grid above the layer that contains the devices. In other words, we have to find vertex disjoint trees with given vertex sets in a 3-dimensional rectangular grid. Clearly, the number of required layers, that is the height of the routing should be optimized. Because of the difficulty of the problem, most of the routing algorithms are approximate and they will not necessarily find the best solutions. As we will see, the problem is not always solvable even with an arbitrary height, moreover even for small instances (e.g. when the circuit board is 2x2) there does not exist a sufficient routing. Consequently, we introduce extra rows and columns to guarantee solvability.

Firstly, we review the main results of the single active layer routing problem (SALRP for short) in the case where both sides of the rectangular board may be extended. In the next section we will study why and when it is necessary to introduce extra rows and columns, and we will prove a result which gives a necessary and sufficient condition for the solvability of the single active layer routing problem without spacing, which seems to be folklore, and we will give an algorithm that realizes the routing itself. Finally, we will consider the case in which the terminals are placed on two layers, so called 3-dimensional channel routing.

2 Basic definitions

Definition 2.1 The vertices of a given planar grid of size $w \times n$ (which consists of w rows and n columns) are called *terminals.* A *net* S is a set of terminals. *A single active layer routing problem* is a set $S = \{S_1, S_2, ..., S_k\}$ of pairwise disjoint nets.

Definition 2.2 By a *spacing of* s_n *in direction* n we are going to mean that we introduce $s_n - 1$ pieces of extra columns between every two consecutive columns (and also on the right hand side of the rightmost column) of the original grid. This way the width of the grid is extended to $n' = s_n n$. A *spacing* of s_w in direction n is defined analogously.

Definition 2.3 A *solution* (or *routing*) with a given s_w and s_n of a single active layer routing problem $S = \{S_1, S_2, ..., S_k\}$ is a set $H = \{H_1, H_2, ..., H_k\}$ of pairwise vertex disjoint, connected subgraphs in the cubic grid of size $(w \cdot s_w) \times (n \cdot s_n) \times h$, such that $S_i \subset V(H_i)$, that is, H_i connects the vertices of S_i. The H_i are called *wires.* h is called the *height* of the routing, and the grids of size $(w \cdot s_w) \times (n \cdot s_n)$ are called *layers.* If $s_w = s_n = 1$ then the problem is solvable *without spacing.*

3 Single Active Layer Routing Problem With Spacing

In this section we study the case where we introduce new rows and columns in order to guarantee solvability. However, in this case it is easy to find a routing, the main problem is to find a routing with height as minimal as possible.

Proposition 3.1 *[2]*

For any given n and s_w there exists a routing problem which cannot be solved with height smaller than $\frac{n}{2s_w}$.

Proof:

Figure 1

Let $w = 2a$, $n = 2b$ and $w' = s_w w$. Consider the following placement of the terminals. Each net consists of only two terminals and they are placed on the circuit board in a symmetrical position. See Figure 1

The number of nets is an. Line e cuts every net into two, so that for any routing with height h must satisfy $w'h \geq an$ which implies $h \geq \frac{n}{2s_w}$. $\square$

Proposition 3.2 *[2]*

If $s_w \geq 2$ and $s_n \geq 2$ than every routing problem can be solved with height $h \leq \frac{nw}{2}$.

Proof: Since 1-terminal nets can be disregarded, the number of nets is at most $\frac{nw}{2}$. We assign a separate layer to each net. With a long h-wire segment we connect each terminal with its layer, then on the layer assigned to the net we introduce a 1-unit w-wire and then a 1-unit n-wire segment. After that, we interconnect the terminals by using only columns and rows that did not belong to the original grid, this guarantees that the long h-wire segments do not intersect the wires on the layers. Hence, the routing is done by using not more than $\frac{nw}{2}$ layers. See Figure 2. $\Box$

Note that if we fix w then if $s_w \geq 2$ and $s_n \geq 2$ then the routing can be realized with height $h = O(n)$, but it is not so obvious if $s_n = 1$ and w is also fixed.

Figure 2

The main results of this area are the following theorems:

Theorem 3.1 *[2]*

If each net consists of two terminals only then a single active layer routing problem can be solved with $s_w = s_n = 2$ *and with height* $h = 3\max\{n, w\}$.

The problem is also solved if we have multiterminal nets, that is each net contains an arbitrary number of terminals:

Theorem 3.2 *[2]*

Any single active layer routing problem can be solved with $s_w = s_n = 2$ *and with height* $h = 6\max\{n, w\}$.

We note here that each theorem mentioned in this section gives a polynomial algorithm for the routing.

4 Single Active Layer Routing Problem Without Spacing

As we have already seen in the previous chapter if we are allowed to extend the width w and the length n then the problem always becomes solvable. In this section we consider the case in which we cannot use extra rows and columns, so that we shall realize the routing above the original grid of size $w \times n$. Let us consider the following single active layer routing problems:

Figure 3

One can easily see that there is no sufficient routing. First of all, we give a necessary and sufficient condition for the solvability of the single active layer problem without spacing, which seems to be folklore. If the problem is solvable we give an algorithm which gives a routing with height $h = O(n^2)$ where $n \geq w$, which is the main result of this paper.

We will call *empty terminals* the terminals which do not belong to any net.

We will use the term *w-wire segment* to refer to a wire segment that is parallel to the width of the grid, and also use *n-wire segment* and *h-wire segment* analogously.

Theorem 4.1 *A single active layer routing problem is solvable without spacing if and only if there exists at least one empty terminal or there are at least* *two neighbored terminals which belong to the same net, and in this case the routing can be realized with height $h = O(n^2)$ where $n \geq w$.*

Proof: Firstly, let us suppose that there is no empty terminal and neither of the neighbored terminals belong to the same net. Therefore, there is an h-wire segment from each terminal, and there is a minimal among them, say belonging to terminal t_1. From the endpoint of this h-wire segment there is an n- or w-wire segment, hence this intersects another wire, but this cannot belong to the net of t_1 because t_1 has no neighbor from its net. Thus, the routing cannot be realized. See Figure 4.

Figure 4

Notice that if there is a wire starting from a terminal, we can say that the terminal is moved to the endpoint of the wire, because if we connect this endpoint with a terminal which belongs to the same net at the same time we connect it with the terminal itself. Now, let us suppose that there are two neighbored terminals. In this case we may connect them on the first layer, and we introduce a wire segment of height 2 from each terminal, so we can consider the second layer as the active layer of a new single active layer routing problem, and we have an empty terminal.

Thus, we have a single active layer routing problem with an empty terminal. We can move this empty terminal, as you can see in Figure 5.

We introduce a new move called *rotation*, which is represented in Figure 6 and in Figure 7.

Figure 5

Figure 6

So, by using an empty terminal we can push a non-empty terminal in a 2x3 rectangle. The height increases by 5 during the rotation.

Figure 7

We will move one of the neighbors of the empty terminal next to an other one which belongs to the same net, the process can be seen in Figure 8. We push the terminal by using the move rotation, and if we have to turn left or right we simply move the empty terminal to the appropriate place.

Note that the height will increase by $O(n)$, because we will have at most $2n$ rotations and each of them increases the height by 5. Then these two terminals are neighbored so we can connect them while the height of the problem is increased by 2. Hence, by increasing the height by $O(n)$ we can make a new empty terminal. After that we do exactly the same $2n$ times and get $2n$ empty terminals with height $O(n^2)$. We would like to note here that by using only rotations and moving empty terminals we can make n^2 empty terminals which means a complete routing. It follows that, the problem is solvable with

Figure 8

height $O(n^3)$, but now we are going to prove that height $h = O(n^2)$ can be achieved.

After that, we move n empty terminals into the first row and n ones in the third row. In the next phase we will wire the nets which have a terminal in the second row in the first, third, fifth etc. column; call these terminals *red terminals* and the corresponding nets red nets. This time we only deal with the red nets, we do not want to interconnect the others. At first we consider the fourth row: if in this row there are some terminals which belong to a red net we can connect them by using the third row which is empty, and we increase the height by only one per connected terminal. If we have connected all terminals we could this way we introduce h-wire segments from every terminal so that the endpoints of the wires are on the same layer.

After that, we make a rotation in the first and in the second columns and in the third and in the fourth column and so on as it can be seen in Figure 9. Black represent the empty terminals, and terminals 2,3,4,5 are the ones that may move during this step, where there is no symbol we can have an arbitrary terminal.

Figure 9

This way we moved the empty rows downwards by 1 and what is really important is that the *red terminals* are still between the two empty rows like they were. Next, we check again the row which is above the lower empty one. If there is a terminal which belongs to a red net we connect them by using the fourth row which is now empty, etc.

We repeat this step n-times and we interconnect the *red nets* and note that this means at least $n/2$ nets, say that this means k_1 connected terminals, because the red nets will surely be connected and so will be the red terminals. Hence, the height has been increased by $O(n) + k_1$.
Now the empty rows are the n^{th} and the $(n-2)^{th}$ ones. Let us consider the first, the third, the fifth terminals of the $(n-1)^{th}$ row. If some of them are empty we can move a non-empty one there by increasing the height by one. Finally, we note that we have wired at least $n/2$ nets and the situation is the same as it was before: we can now introduce new red terminals and iterate this procedure described above.
If we iterate this procedure n times, all of the nets will be interconnected and we get that the height of the routing is $O(n^2) + \sum k_i = O(n^2)$, since $\sum k_i$ is the number of the terminals. $\square$

5 3-Dimensional Channel Routing

After having solved the single active layer routing problem we may ask the question what if the terminals are not on a single layer. In this chapter we will study the case where the terminals are placed on the bottom and on the top layer. The object of the problem is the same as it was in the single active layer routing problem: to connect the terminals in each net with Steiner-trees in the 3-dimensional cubic grid by using as few layers as possible in such a way that Steiner-trees which interconnect distinct nets are vertex-disjoint.

If we are allowed to introduce extra rows and columns (in both grid), then the problem is always solvable, moreover:

Theorem 5.1 *[4]*

Every 3-dimensional channel routing problem can be solved with $s_w = s_n = 2$ *and height* $h = 15\max\{n, w\}$.

From now on we will consider the 3-dimensional channel routing problem without spacing.

Hereafter, we will disregard the case where each net consists of only two terminals which are opposite because in this particular case we can trivially solve the problem by introducing long h-wire segments from each terminal.

Note that if the 3-dimensional channel routing is solvable then if we consider the bottom (or the top layer) as the active layer of a single active layer routing problem then this is also solvable.

Now we prove the other new result of this paper.

Theorem 5.2 *If the bottom layer contains at least two empty terminals and the top layer contains an empty terminal or two neighbored terminals of the same net then the 3-dimensional channel routing problem is solvable with height* $h = O(n^3)$ *where* $n \geq w$.

To prove this proposition first we define $(w \times n - 2)$ puzzles. This is closely related to the single active layer routing and the channel routing as well. The $(w \times n - 2)$-puzzle is a generalization of Sam Loyd's well-known 15-puzzle. Sam Loyd's puzzle is the following: we panel a square board into 4×4 squares and we put on the board 15 numbered tiles, see Figure 10

Figure 10

Our aim is to reverse the 14^{th} and 15^{th} tile by using the following move: we can exchange the nonblank tile with its neighbored blank tile. However, in the $w \times n - 2$ puzzle we have a $w \times n$ table with two blank tiles and the allowed moves are exactly the same.

This $w \times n$-puzzle is naturally associated with a single active layer routing or a 3-dimensional channel routing problem as follows. If we look at the cubic grid from above, on the top layer we can see the placement of the terminals. If we only move the empty terminal several times and after every move we look at the cubic grid from above, we remark that this is like a $w \times n$-puzzle. This association can be seen in Figure 11

Figure 11

Now let us consider a 3-dimensional channel routing problem with the condition written in the theorem. Firstly, consider the problem as two single active layer routing problems. Since the solvability is guaranteed we can solve each of them with height $h = O(n^2)$ (Note that it is possible that each net has exactly one terminal on the bottom layer and exactly one terminal on the top layer, in this case we have not introduced any wires). Hence, we have two layers and none of them contains more then one terminal which corresponds to the same net. Furthermore, on the lower one there are at least two empty terminals and on the upper one there is at least one empty terminal.

Now we want to get terminals that belong to the same net in the same row and column so that we would connect them with a single long h-wire segment.

Consider the placement of the terminals as two $w \times n$-puzzles. We want to see the very same configuration on them.

Let C_1 and C_2 be arbitrary configurations. Let π be a permutation on the nonblank tiles such that $\pi(t_1) = t_2$ if and only if the location of a nonblank tile t_1 is the same as the location of a nonblank tile t_2 in C_2. Wilson showed [3] that C_1 is reachable from C_2 if and only if π is an even permutation (that is, the number of inversions is even).

Furthermore, Parberry showed [5] that we can reach the configuration with $O(n^3)$ moves, which means $h = O(n^3)$.

Since by increasing the height with $O(n)$ we can move an empty terminal next to the other one, we can have two blank tiles in a 2×2 square. In this square we can change the inversion number by one. Thus in this case π can be even so we can reach the same configuration on the two layers, so that with a long h-wire segment we can interconnect the terminals, and we have used $O(n^3)$ layers, which ends our proof. $\square$

Note that our proof implies that the 3-dimensional channel routing is not solvable if and only if we have at most one empty terminal on the bottom and at most one empty terminal on the top layer and permutation π (which is defined as above) is odd.

We note here that the routing cannot be realized with height $h = O(n^2)$. Let us consider the following channel routing problem: the bottom and the top layer is $n \times n$. On the bottom layer the terminals $1, 2 ... n^2 - 2$ are one after the other, from left to right and from up to down, and if we rotate this layer about its center with $\pi/2$ we get the placement of the terminals on the top layer. In this case we have $O(n^2)$ terminals and the distance between two terminals that belong to the same net is $O(n)$, thus we need at least $O(n^3)$ layers.

6 Summary

As we have seen in this paper some 3-dimensional VLSI routing problems can be handled by combinatorial algorithms. If we are allowed to extend the length and the width of the problem, there always exists a routing. The main problem is to use as few layers as possible. In the area that I have studied we do not use any spacing so the problem is not always solvable. I gave a necessary and sufficient condition for the solvability of the single active layer routing without spacing, and an algorithm that realizes the routing with height $h = O(n^2)$. We have seen the connection between 3-dimensional channel routing and puzzles, and I proved that in certain cases the routing can be realized with height $h = O(n^3)$.

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